Mathematical Appendix Chapter A1

A1.2 Just use the definitions of subsets, unions, and intersections.

A1.3 To get you started, consider the first one. Pick any $x \in (S \cap T)^c$. If $x \in (S \cap T)^c$, then $x \notin S \cap T$. If $x \notin S \cap T$, then $x \notin S$ or $x \notin T$. (Remember, this is the inclusive "or".) If $x \notin S$, then $x \in S^c$. If $x \notin T$, then $x \in T^c$. Because $x \in S^c$ or $x \in T^c$, $x \in S^c \cup T^c$. Because x was chosen arbitrarily, what we have established holds for all $x \in (S \cap T)^c$. Thus, $x \in (S \cap T)^c \Rightarrow x \in S^c \cup T^c$, and we have shown that $(S \cap T)^c \subset S^c \cup T^c$. To complete the proof of the first law, you must now show that $S^c \cup T^c \subset (S \cap T)^c$.

A1.13 To get you started, let $x \in f^{-1}(B^c)$. By definition of the inverse image, $x \in D$ and $f(x) \in B^c$. By definition of the complement of B in R, $x \in D$ and $f(x) \notin B$. Again, by the definition of the inverse image, $x \in D$ and $x \notin f^{-1}(B)$. By the definition of the complement of $f^{-1}(B)$ in $D, x \in D$ and $x \in (f^{-1}(B))^c$, so $f^{-1}(B^c) \subset (f^{-1}(B))^c$. Complete the proof.

A1.18 Let $\frown^i = \{\mathbf{x} | \mathbf{a}^i \cdot \mathbf{x} + b^i \geq 0\}$. Use part (b) of Exercise A1.17.

A1.21 First, model your proof after the one for part 3. Then consider $\cap_{i=1}^{\infty} A_i$, where $A_i = (-1/i, 1/i)$.

A1.22 Draw a picture first.

A1.24 Look at the complement of each set.

A1.25 Use Theorem A1.2 to characterize the complement of S in R.

A1.26 For the first part, sketch something similar to Fig. A1.12 and use what you learned in Exercise A1.24. The second part is easy.

A1.27 To get you started, note that the complement of S is open, then apply Theorem A1.3. Open balls in R are open intervals. Use what you learned in Exercise A1.26.

A1.31 Center a ball at the origin.

A1.32 For part (c), you must show it is bounded *and* closed. For the former, center a ball at the origin. For the latter, define the sets $F_0 \equiv \{\mathbf{x} \in \mathrm{R}^n | \sum_{i=1}^n x_i = 1\}$, $F_i \equiv \{\mathbf{x} \in \mathrm{R}^n | x_i \geq 0\}$, for $i = 1, \cdots, n$. Convince yourself that the complement of each set is open. Note that $S^{n-1} = \cap_{i=0}^n F_i$. Put it together.

A1.38 Look closely at S.

A1.39 Check the image of $f(x) = \cos(x) - 1/2$.

A1.40 Choose a value for y, some values for x_1, and solve for the values of x_2. Plot x_1 and x_2.

A1.46 In (b), it may help to remember that $\mathbf{x}^1$ and $\mathbf{x}^2$ can be labeled so that $f(\mathbf{x}^1) \geq f(\mathbf{x}^2)$, and that $tf(\mathbf{x}^1) + (1-t)f(\mathbf{x}^2) = f(\mathbf{x}^2) + t(f(\mathbf{x}^1) - f(\mathbf{x}^2))$.

A1.49 Yes, yes, no, yes. Look for convex sets. For (e), things will be a bit different if you assume $f(x)$ is continuous and if you don't.